

TOPIC 2

Math Modeling in 3 Acts:
Swift Kick



Lesson Overview

Mathematics Objectives

- ✓ Use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ Test and verify the appropriateness of their math models.
- ✓ Explain why the results from their mathematical models might not align exactly with the problem situation.

Language Objective

- ✓ Use precise mathematical language with peers to discuss strategies and identify problem-solving methods.

Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model might not represent the real-world situation exactly.

Earlier in this topic, students:

- Solved systems of equations.
- Explored different forms of quadratic functions.

In this lesson, students:

- Develop a mathematical model to represent and propose a solution to a problem situation involving quadratic functions.

Later in this topic, students will:

- Refine their mathematical modeling skills by solving quadratic equations and linear-quadratic systems.

This mathematical modeling lesson focuses on application of both math content and math processes.

- Students draw on their understanding of concepts related to quadratic functions to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

MATH
MODELING
IN 3 ACTS

Mathematical Process
Standards
2A.1A Apply mathematics to
problems arising in everyday
life, society, and the workplace.
Also 2A.1B, 2A.1E, and 2A.1G

TEKS
2A.4A, 2A.8C

Swift Kick

Whether you call it soccer, football, or fútbol, it's the most popular sport in the world by far. Even if you don't play soccer, you probably know several people who do.

There are many ways to kick a soccer ball: you can use any part of either foot. If you want the ball to end up in the goal, you also need to try different amounts of spin and power. You'll see one person's effort in the Math Modeling in 3-Acts lesson.

ACT 1

Identify the Problem

1. What is the first question that comes to mind after watching the video?
2. Write down the main question you will answer about what you saw in the video.
3. Make an initial conjecture that answers this main question.
4. Explain how you arrived at your conjecture.
5. What information will be useful to know to answer the main question? How can you get it? How will you use that information?

ACT 2

Develop a Model

6. Use the math that you have learned in this Topic to refine your conjecture.

ACT 3

Interpret the Results

7. Did your refined conjecture match the actual answer exactly? If not, what might explain the difference?

BUILD G.R.I.T. Resilience
It is okay to struggle. If one
approach to finding a model
is not working, try a different
approach.

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TOPIC 2 Math Modeling in 3 Acts

Student Companion

Students can do their work for the task on their *Student Companion* pages, which you can print from [SavvasRealize.com](https://www.savvasrealize.com).

Texas Essential Knowledge and Skills (TEKS)

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

2A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

2A.1A apply mathematics to problems arising in everyday life, Number Flexibility to 100 for All Four Operations

2A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Also **2A.1E**, **2A.1F**, **2A.1G**

Quadratic and square root functions, equations, and inequalities. The student applies mathematical processes to understand that quadratic and square root functions, equations, and quadratic inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:

2A.4A write the quadratic function given three specified points in the plane Data. The student applies mathematical processes to analyze data, select appropriate models, write corresponding functions, and make predictions.

2A.8C predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models.



Swift Kick

In this mathematical modeling task, students analyze partial views of soccer shots to determine whether the attempts are successful. They need to determine whether the ball's path will pass through the goal. To do so, they apply concepts they study in Topics 1 and 2.

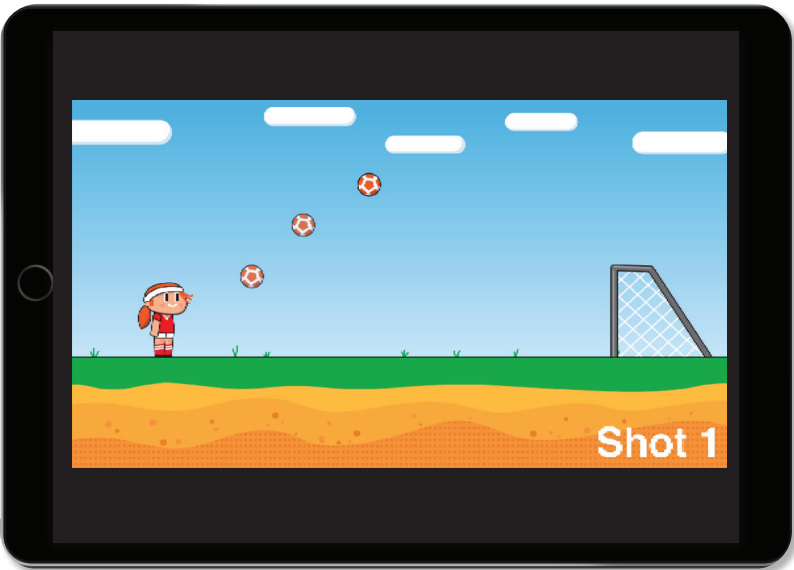
ACT 1 The Hook

Play the video. The video shows a video game character shooting soccer balls in profile. Each shot traces part of a parabola. The video does not show the result of any of the attempts.

After the question brainstorming, have students consider the Main Question. Remind students to write down their questions and conjectures.

MAIN QUESTION

Which of the shots go in?



ACT 2 Modeling With Math

Think about the task. Ask students to speculate how they can determine which shots go in. Then have them think about what information they need.

Reveal the information. Use the video, which includes a coordinate grid and sample points for each shot, to give students specific information about the paths. Then, you can use the graphing calculator to explore quadratic functions that model each shot.

What's the connection? Give students time to struggle as they think about how to connect their ideas to what they learned in this topic about quadratic functions. How could they use the information to write a function for each shot? How can they use those functions to solve the problem?

Before showing the answers, find out where there is disagreement. Allow students to argue their cases and update their conjectures.

INTERESTING MOMENTS WITH STUDENTS

Students may assume that every curve includes the vertex. Shots 3 and 4 stop at the vertex, but the other shots stop before the vertex. Students may incorrectly think they can use symmetry for all curves to determine the paths of the shots. Invite a student who uses this method to explain it during Act 3. This may lead to a conversation about how Shot 4 is a counterexample to this conjecture.

Necessary Information

Horizontal distance: 36 ft

Height of goal: 8 ft

Coordinates

- Shot 1: (5, 7) and (15, 15)
- Shot 2: (6, 2.8) and (15, 6.25)
- Shot 3: (10, 13.75) and (16, 16)
- Shot 4: (11, 8.25) and (22, 11)
- Shot 5: (3, 2.5) and (12, 8)



ACT 3 The Solution

Play the video. The final video shows the entirety of the five shots. The outcome of each shot is revealed, including an overhead angle. After each shot, ask students for a show of hands of whose initial conjecture was accurate.

MAIN QUESTION ANSWER

- Shot 1: The ball goes in the top-right corner.
- Shot 2: The ball misses high.
- Shot 3: The ball goes in on one bounce.
- Shot 4: The ball is the right height but misses left.
- Shot 5: The ball misses. It hits the crossbar.

Do the “post-game” analysis. Help students understand why a shot’s result may not match what the parabola predicts. Students should focus on how the paths are in 2-D and do not show the third dimension. They may not have considered that Shot 3 could bounce in or that Shot 4 has an accurate path from one perspective but not from another. Look for creative ideas students offer to improve the method for predicting whether a shot goes in.

Now that the solution has been revealed and discussed, ask students to analyze and evaluate the efficiency of the approach they chose to solve the problem.

ONE POSSIBLE SOLUTION

For each shot, substitute the two given points (x, y) and the origin into the standard form of a parabola. Start with Shot 1.

Use (0, 0).	Use (5, 7).	Use (15, 15).
$y = ax^2 + bx + c$	$y = ax^2 + bx + c$	$y = ax^2 + bx + c$
$0 = a(0)^2 + b(0) + c$	$7 = a(5)^2 + b(5) + c$	$15 = a(15)^2 + b(15) + c$
$0 = c$	$7 = 25a + 5b + c$	$15 = 225a + 15b + c$

Since $c = 0$, the result is a system of two linear equations.

$25a + 5b = 7$
 $225a + 15b = 15$

The solution of the system is $a = -0.04$, $b = 1.6$.

Now, substitute a , b , and c into standard form.

$y = -0.04x^2 + 1.6x$

Finally, substitute 36 for x .

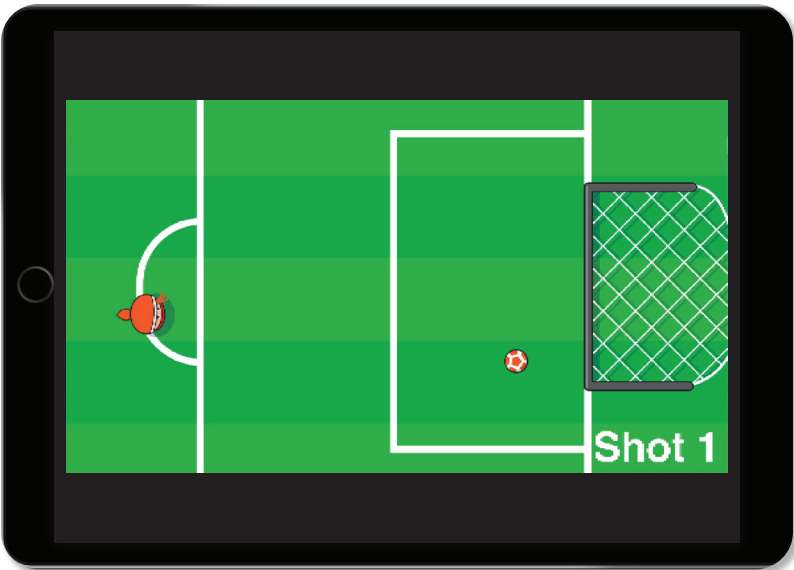
$y = -0.04(36)^2 + 1.6(36)$
 $y = 5.76$

The height of the ball when it reaches the goal is 5.76 ft.

Since $5.76 < 8$, the shot will go in the goal.

Repeat for Shots 2–5.

- Shot 2: $y = -0.005x^2 + 0.5x$; when $x = 36$, $y = 10.8$; shot misses
- Shot 3: $y = -0.0625x^2 + 2x$; when $x = 36$, $y = -9$; shot goes in
- Shot 4: $y = -0.0227x^2 + x$; when $x = 36$, $y \approx 6.55$; shot goes in
- Shot 5: $y = -0.0185x^2 + 0.8x$; when $x = 36$, $y = 8$; shot misses



INTERESTING MOMENTS WITH STUDENTS

Shots 3, 4, and 5 are designed to start a discussion. Students can discuss how the ground, the third dimension, and the boundaries of the goal influence the result. For example, students may think that a shot that passes through (36, 8) will barely go in when it actually hits the crossbar of the goal.

CREATE A PROBLEM

As students finish, have them construct their own problem involving a projectile. Challenge students to require finding the solutions to a linear-quadratic solution or a quadratic inequality. Then have them trade with a partner to solve.